

MLR Interactions and variable transformations

Author: Nicholas G Reich, Jeff Goldsmith

*This material is part of the **statsTeachR** project*

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Today's Lecture

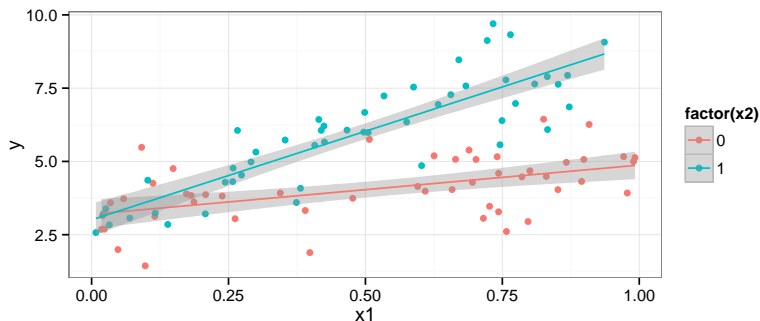
A few important building blocks for regression

- Interaction models
- Transformations of predictors

What is interaction?

Definition of interaction

Interaction occurs when the relationship between two variables depends on the value of a third variable.



[Good overview: KNN pp. 306–313]

Some real world examples?

How to include interaction in a MLR

$$\text{Model A: } y_i = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \epsilon_i$$

$$\text{Model B: } y_i = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \beta_3 x_{i1} \cdot x_{i2} + \epsilon_i$$

Key points

- “easily” conceptualized with 1 continuous, 1 categorical variable
- models possible with other variable combinations, but interpretation/visualization harder
- two variable interactions are considered “first-order” interactions (often used to define a class of models)
- still a **linear** model, but no longer a strictly **additive** model

How to interpret an interaction model

For now, assume x_1 is continuous, x_2 is 0/1 binary.

$$\text{Model A: } y_i = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \epsilon_i$$

$$\text{Model B: } y_i = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \beta_3 x_{i1} \cdot x_{i2} + \epsilon_i$$

Example interaction model with FEV data

$$fev_i = \beta_0 + \beta_1 age_i + \beta_2 ht_i + \beta_3 sex_i + \beta_4 smoke_i + \beta_5 ht \cdot smoke_i + \epsilon_i$$

```
mi1 <- lm(fev ~ age + ht + sex + smoke, data = dat)
mi3 <- lm(fev ~ age + ht * smoke + sex, data = dat)
c(AIC(mi1), AIC(mi3))
```

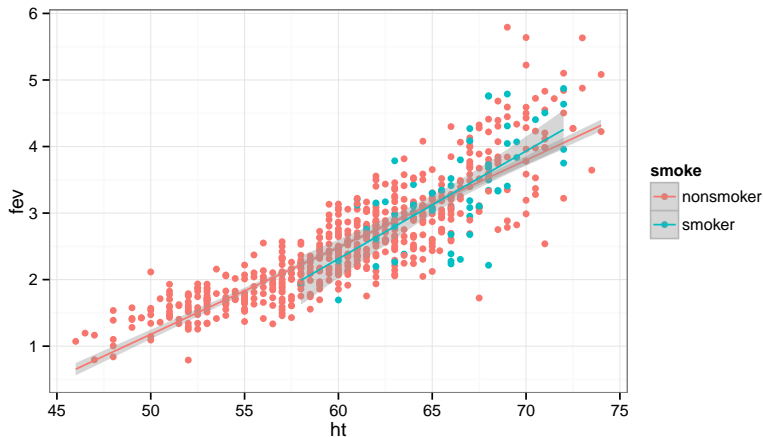
```
## [1] 703.8 700.5
```

```
round(summary(mi3)$coef, 2)
```

| ## | Estimate | Std. Error | t value | Pr(> t) |
|-------------------|----------|------------|---------|----------|
| ## (Intercept) | -4.35 | 0.23 | -19.12 | 0.00 |
| ## age | 0.07 | 0.01 | 7.17 | 0.00 |
| ## ht | 0.10 | 0.00 | 21.08 | 0.00 |
| ## smokesmoker | -2.61 | 1.10 | -2.37 | 0.02 |
| ## sexmale | 0.15 | 0.03 | 4.43 | 0.00 |
| ## ht:smokesmoker | 0.04 | 0.02 | 2.30 | 0.02 |

Example interaction model with FEV data

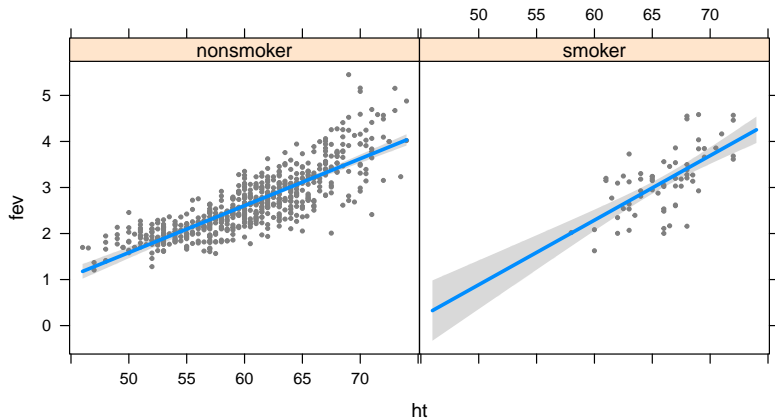
```
qplot(ht, fev, data=dat, color=smoke,  
       geom=c("point", "smooth"), method="lm")
```



Example interaction model with FEV data

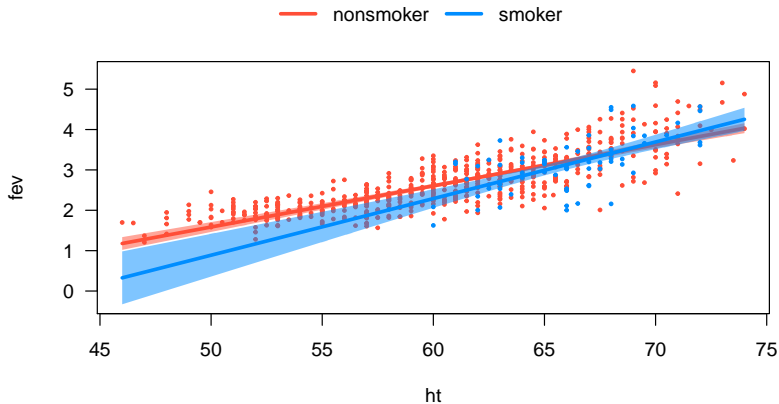
The visreg package plots not the data but the partial residuals (a.k.a. the adjusted variable) plot.

```
require(visreg)
visreg(mi3, "ht", by = "smoke")
```



Example interaction model with FEV data

```
visreg(mi3, "ht", by = "smoke", overlay = TRUE)
```



Overview of variable transformations

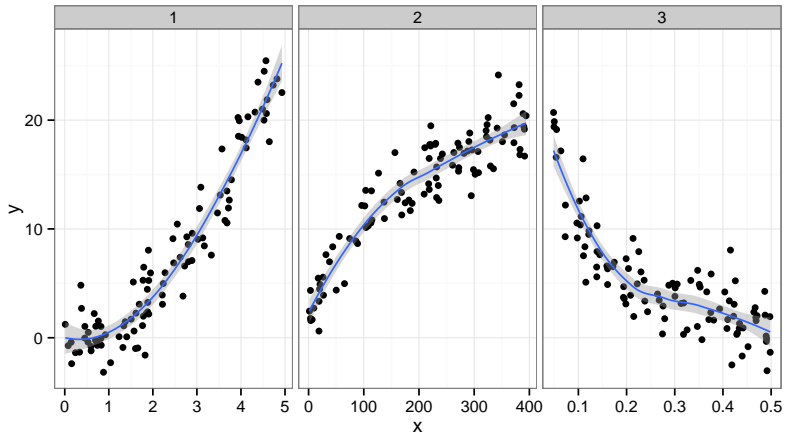
The problems

- Non-linearity between X and Y \longrightarrow transform X
- Skewed distribution of X s/points with high leverage \longrightarrow transform X
- Non-constant variance \longrightarrow transform Y

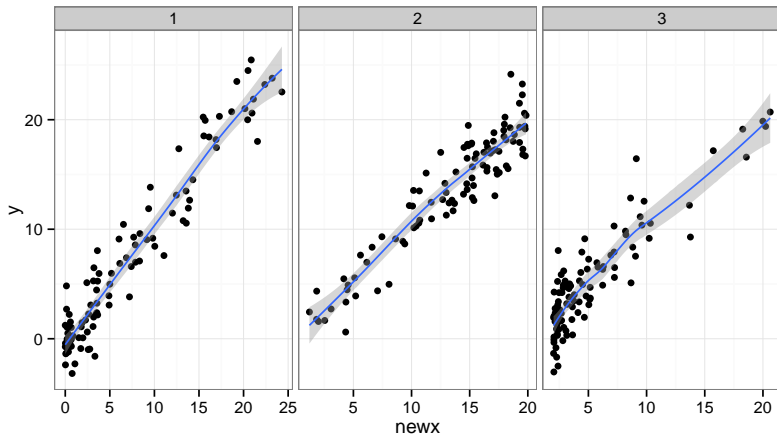
[More info: KNN Ch 3.9, pp. 129–137]

Transforming your X variables

Transforming predictor variables can help with constant-variance non-linear relationships.



Transforming your X variables



β interpretations with transformed X s

Transforming predictor variables can help with non-linearities, but can make coefficient interpretations hard.

Possible solutions

- Interpret β s qualitatively across a region of interest: “We found strong evidence for an inverse association, where values of Y decreased inversely proportional to X across the observed range (a, b) .”
- Occasionally, a “one unit change in X ” can be meaningful: e.g. $\log_a X$. A one unit change in $\log_a X$ indicates a a -fold increase in X .

β interpretations with transformed X s

Transforming predictor variables can help with non-linearities, but can make coefficient interpretations hard.

Transforming Y s for non-constant variance

What to do ...

- Nothing; just use least squares and bootstrap
- Use weighted LS, GLS (Methods 3?)
- Use a variance stabilizing transformation
- Consider a generalized linear model (more soon)

Box-Cox Transformations

Outcome is raised to the λ power:

$$y_i^\lambda = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \epsilon_i$$

- Estimate λ , a new parameter, by maximum likelihood.
- Some well-known choices of λ : 2, -1, 1/2
- By definition, when $\lambda = 0$, we specify $y_i^\lambda = \log_e y_i$

[More detailed info: KNN Ch 3.9, pp. 134–137]

Wrap-up

New instruments for your regression tool-kit

- Interactions and data transformations are common extensions/additions to regression models in practice.
- Both are simple to implement, challenging to interpret correctly!
- But, you may not always need an interpretation, e.g. you might just want a good prediction.